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Nonperturbative Corrections
in Resummed Cross Sections

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Abstract

We show that the resummation of large perturbative corrections in QCD leads to ambiguities in high energy cross sections that are suppressed by powers of large momentum scales. These ambiguities are caused by infrared renormalons, which are a general feature of resummed hard-scattering functions in perturbative QCD, even though these functions are infrared safe order-by-order in perturbation theory. As in the case of the operator product expansion, the contributions of infrared renormalons to coefficient functions may be absorbed into the definition of higher-dimensional operators, which induce nonperturbative corrections that are power-suppressed at high energies. The strength of the suppression is determined by the location of the dominant infrared renormalon, which may be identified explicitly in the resummed series. In contrast to the operator product expansion, however, the relevant operators in factorized hadron-hadron scattering and jet cross sections are generally nonlocal in QCD, although they may be expressed as local operators in an effective theory for eikonalized quarks. In this context, we verify and interpret the presence of $1/Q$ corrections to the inclusive Drell-Yan cross section with Q the pair mass. In a similar manner, we find $\exp(-b^2 \ln Q)$ corrections in the impact parameter space of the transverse momentum distributions of the Drell-Yan process and e^+e^- annihilation. We also show that the dominant nonperturbative corrections to cone-based jet cross sections behave as $1/(Q\delta)$, with δ the opening angle of the jet and Q the center of mass energy.

1. Introduction

It is almost axiomatic that perturbation theory alone cannot give a full description of QCD. Luckily, however, perturbative calculations may be supplemented by nonperturbative functions and parameters to derive physical predictions. While other sources of nonperturbative behavior are possible, it is attractive to identify those required to make perturbation theory well defined. In this manner, we may use perturbation theory as a diagonstic tool to identify a minimal set of nonperturbative parameters. This is the viewpoint that we shall advocate and exploit below. It is of particular interest for infrared safe quantities, which are finite order-by-order in perturbation theory. For such quantities, we expect perturbation theory to predict leading behavior as an asymptotic expansion in the coupling, and for the failure of perturbation theory at high orders to be particularly enlightening.

To be specific, we shall work in the class of quantities for which leading logarithmic behavior in energy is described by Sudakov resummation [1, 2, 3, 4, 5]. This resummation may be formulated in terms of path-ordered exponentials, or Wilson lines. Path-ordered exponentials have been extensively studied in perturbation theory [6, 4] and are also of special interest for a nonperturbative description of the theory. We will see below that they naturally relate so-called infrared renormalons [7, 8, 9] to nonlocal operator expectation values [10] for cross sections of lepton pair production [11, 12] and energy-energy correlations [2] in e^+e^- annihilation. Generalizations to other quantities, including jet cross sections, and to nonleading logarithms are also discussed.

Already in [13] it was shown that resummation of large perturbative corrections to the inclusive lepton pair cross section leads to an ambiguity in perturbation theory at the level of Λ/Q, with Q the pair mass and Λ the QCD scale, compared to leading power. This paper is in large part an attempt to further understand such power-suppressed effects[1]. We shall identify the corrections implied by these ambiguities with specific nonlocal operators below. In a similar manner, we shall find $\exp(-b^2 \ln Q)$ corrections (previously-discussed in [2]) in the impact parameter space for the transverse momentum distribution in the Drell-Yan cross section and in e^+e^- annihilation. In addition, we shall see that nonperturbative corrections to cone-based jet cross sections may be expected to begin at $1/(Q\delta)$, with δ the opening angle of the jet and Q the center of mass energy. An application to inclusive B meson decay has been given in [15].

Unaided, perturbation theory fails in a number of ways. At low orders, infrared divergences and long-distance dependence at fixed orders in perturbation theory can be factorized into nonperturbative functions like parton distributions, which are determined by experiment [16]. In addition, QCD perturbation theory is (probably) at best an asymptotic expansion for Green functions, even off the mass shell. Relevant for us are individual diagrams that behave at n loops as $\alpha_s^n n!$. A series that behaves in this fashion is not Borel summable, let alone convergent, because such terms produce singularities in the Borel transform that prevent it from being inverted in a unique fashion. Singularities that are associated with individual diagrams are often termed infrared or ultraviolet renormalons. In QCD, it is the infrared (IR) renormalons that directly interfere with the inversion of the Borel transform [7, 8].

A lack of Borel summability does not imply that a series is meaningless, however. Rather, it may represent an asymptotic expansion for a set of functions that differ only in ways that do not affect this expansion. For instance, they may differ in powers of $\exp[-1/\alpha]$, with α the expansion parameter. Different members of this class of functions correspond to various definitions of the

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[1]For recent related studies see Ref. [14].

inverse Borel transform. It is this sort of ambiguity with which we shall deal below, showing how its resolution requires the introduction of new, nonperturbative, parameters associated with vacuum expectations for nonlocal operators[2].

The paper is organized as follows. In Sect. 2 we review how infrared renormalons arise in resummed cross sections. Analyzing their contributions to a Wilson line expectation value, we then find a general form for power suppressed corrections associated with infrared renormalons in the transverse momentum distribution of lepton pairs. In Sect. 3 we relate the contribution of the leading infrared renormalon to the matrix element of a nonlocal gluon field operator, which can also be represented as a local operator in an effective field theory for eikonal quarks. In Sects. 4 and 5 we consider power corrections to jet cross sections and inclusive lepton pair production, and identify in each case a leading infrared renormalon that results in Λ/Q corrections. Sect. 6 contains concluding remarks.

2.   Infrared renormalons in a resummed cross section

In many applications of perturbative QCD to high energy scattering, it is desirable to sum finite corrections that remain after the cancellation of infrared divergences. Examples are found in energy-energy correlations in e^+e^- annihilation [2, 18], the transverse momentum distribution of lepton pairs [19, 11], Sudakov effects in elastic scattering [20], the total cross section for lepton pair production [21, 22, 23, 24, 13] and for heavy quark production [25], and also in heavy quark effective theory [26, 15]. As we shall see, such resummations of perturbative corrections generally imply the presence of nonperturbative corrections suppressed by powers of kinematic invariants.

Let us first consider the transverse momentum distribution of lepton pairs in the Drell-Yan process, $h + h' \to \ell^+\ell^- + X$. The relevant formalism is virtually identical for energy-energy correlations in e^+e^- annihilation, closely related to jet structure [2]. For large transverse momentum of the lepton pair, $Q_t \sim Q$, the differential cross-section for the process may be written to leading power in the lepton invariant mass Q as

$$\frac{\mathrm{d}\sigma_{hh'}}{\mathrm{d}Q^2\mathrm{d}Q_t^2} = \sigma_0 \Sigma(Q^2, Q_t^2)\,, \tag{1}$$

where $\sigma_0 = 4\pi^2\alpha^2/9Q^2s$ is the inclusive Born cross section for a quark of unit charge, where s is the squared invariant mass of the incoming hadrons, and where the function Σ depends on parton distributions $f_{a/h}(x, \mu^2)$ through the factorization formula [16],

$$\Sigma(Q^2, Q_t^2) = \int_0^1 \frac{\mathrm{d}x_a}{x_a}\frac{\mathrm{d}x_b}{x_b} f_{a/h}(x_a, Q^2) f_{b/h'}(x_b, Q^2) H_{ab}\left(\frac{Q^2}{x_a x_b s}, \frac{Q_t^2}{Q^2}\right)\,. \tag{2}$$

The bulk of the lepton-pair cross section, however, is in the region $Q_t^2 \ll Q^2$. In this region the function Σ contains "double-logarithmic" corrections like $1/Q_t^2 \times \alpha_s^n \ln^{2n-1}(Q_t^2/Q^2)$. The resummation of these large corrections is most easily expressed in the Fourier **b**-space conjugate to $\mathbf{Q}_t$,

$$\Sigma(Q^2, Q_t^2) = \int \frac{\mathrm{d}^2b}{(2\pi)^2}\ e^{i\mathbf{Q}_t\cdot\mathbf{b}}\ \tilde{\Sigma}(Q^2, b^2)\,, \tag{3}$$

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[2]An application of similar ideas to QED is given in [17].

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where $b^2 \equiv \mathbf{b}^2$. Then, the double-logarithmic corrections to $\tilde{\Sigma}(Q^2, b^2)$ are of the form $\alpha_s^n \ln^{2n} Q^2 b^2$ and exponentiate. At leading power in Q^2, the expression for $\tilde{\Sigma}(Q^2, b^2)$ is given by [2, 11]

$$\tilde{\Sigma}(Q^2, b^2) = \int_0^1 \frac{\mathrm{d}x_a}{x_a} \frac{\mathrm{d}x_b}{x_b} f_{a/h}(x_a, C/b^2) f_{b/h'}(x_b, C/b^2) \delta(1 - Q^2/x_a x_b s) \, \mathrm{e}^{-S(Q^2, b^2)}, \qquad (4)$$

where $\mathrm{e}^{-S(Q^2, b^2)}$ is a resummation factor that includes the large logarithms in $Q^2 b^2$,

$$S(Q^2, b^2) = \int_{C/b^2}^{Q^2} \frac{\mathrm{d}k_t^2}{k_t^2} \left[\ln \frac{Q^2}{k_t^2} \Gamma_{\mathrm{cusp}}(\alpha_s(k_t^2)) + \Gamma(\alpha_s(k_t^2)) \right], \qquad C = 4\mathrm{e}^{-2\gamma}, \qquad (5)$$

with γ the Euler constant. Here, $\Gamma_{\mathrm{cusp}}(\alpha_s)$ is the universal "cusp" anomalous dimension [4, 5], while $\Gamma(\alpha_s)$, which contributes to nonleading behavior in $\ln Q$, depends on the specific process and choice of the constant C. With the choice above, both anomalous dimensions begin at order α_s,

$$\Gamma_{\mathrm{cusp}}(\alpha_s) = \frac{\alpha_s}{\pi} C_F + O(\alpha_s^2), \qquad \Gamma(\alpha_s) = -\frac{3}{2} \frac{\alpha_s}{\pi} C_F + O(\alpha_s^2), \qquad (6)$$

with $C_F = 4/3$. A short calculation, using the one-loop form of the running coupling,

$$\alpha_s(k_t^2) = \frac{1}{\beta_1 \ln(k_t^2/\Lambda^2)}, \qquad \beta_1 = (11 - 2n_f/3)/(4\pi), \qquad (7)$$

shows that the leading (double) logarithms of $Q^2 b^2$ found order-by-order are replaced in the resummed form by

$$S(Q^2, b^2) = \frac{C_F}{\pi \beta_1} \left[\ln(Q^2/\Lambda^2) \ln \frac{\ln(Q^2/\Lambda^2)}{\ln(C/b^2 \Lambda^2)} - \ln(Q^2 b^2/C) \right], \qquad (8)$$

where we have used the one-loop expression (6) for Γ_{cusp} and have neglected nonleading logarithmic terms due to Γ in (5). At large b, the Fourier transform $\tilde{\Sigma}(Q^2, b^2)$ is strongly suppressed by the resummed exponent, which makes the structure function $\Sigma(Q^2, Q_t^2)$ amenable to perturbative treatment. We also note, however, that the exponent diverges at $b^2 = C/\Lambda^2$, because of the singularity in the running coupling (7) at $k_t^2 = \Lambda^2$. It is the consequences of this singularity for power corrections in b^2 that we shall discuss below. First of all, we notice that approaching the "critical" value of the impact parameter $b \sim 1/\Lambda$, one has to take into account power corrections $\mathcal{O}(\Lambda^2 b^2)$, which are negligible with respect to logarithmic corrections $\ln(\Lambda^2 b^2)$ for small b^2, but which become important for large b^2. We stress that the factorization in (4) was found in the leading $1/Q^2$ limit and therefore in principle includes power corrections of $\mathcal{O}(\Lambda^2 b^2)$ provided that $b^2 Q^2 \gg 1$, or equivalently $Q_t^2 \ll Q^2$. At the same time, the expression (5) was obtained by summing all logarithmic corrections and neglecting power corrections. Thus, to study the b^2 power corrections we have to find an approximation for the exponent $S(Q^2, b^2)$ that includes both logarithmic and power corrections in b^2. To this end, we recall the origin of $\mathrm{e}^{-S(Q^2, b^2)}$ in the factorized expression (4).

In the Drell-Yan process, the resummation factor, $\mathrm{e}^{-S(Q^2, b^2)}$, appears as the contribution of real and virtual soft gluons interacting with the quark and antiquark that annihilate to produce a virtual photon. The interaction of soft gluons with quarks can be treated using the eikonal approximation and can be summarized in terms of path-ordered exponentials (Wilson lines) [24]

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as follows. Let us introduce a notation for "straight" path-ordered exponentials in the direction of momentum p,

$$\Phi_p[Sp + x, S_0 p + x] = \mathrm{P} \exp\left(-ig \int_{S_0}^{S} \mathrm{d}s\, p^\mu A_\mu(ps + x)\right),\qquad(9)$$

where P denotes ordering in group indices of the gluon field $A_\mu(x)$. Similarly, for products of two such ordered exponentials, defined by the light-like momenta p_1 and p_2 of the incoming quark and antiquark, respectively, we define

$$U_{\mathrm{DY}}(x) = \mathrm{T}\left[\Phi_{p_2}^\dagger(x, -\infty)\, \Phi_{p_1}(x, -\infty)\right] = \mathrm{T}\left[\Phi_{-p_2}(-\infty, x)\, \Phi_{p_1}(x, -\infty)\right],\qquad p_1^2 = p_2^2 = 0\,,$$
$$(10)$$

where T is time-ordering. Finally, in terms of such path-ordered exponentials, we define the eikonalised Drell-Yan cross-section as [24]

$$W_{\mathrm{DY}}(b, \mu) = \langle 0|\, U_{\mathrm{DY}}^\dagger(0)\, U_{\mathrm{DY}}(b)\, |0\rangle\,,\qquad b^\nu = (0, 0, \mathbf{b})\qquad(11)$$

with $b_\nu b^\nu \equiv -b^2$. If we insert a complete set of states, $1 = \sum_X |X\rangle\langle X|$ between the operators U and $U^\dagger$,

$$W_{\mathrm{DY}}(b, \mu) = \sum_X |\langle 0|\, U_{\mathrm{DY}}(0)\, |X\rangle|^2 \exp(-i\mathbf{k}_t \cdot \mathbf{b})\,,\qquad(12)$$

the Wilson line expectation value W_{DY} describes the cross section for the emission of gluons of any energy by two eikonalized quarks, weighted by a factor $\exp(-i\mathbf{k}_t \cdot \mathbf{b})$, with $\mathbf{k}_t$ the total transverse momentum of gluons in the final state $|X\rangle$. Such a cross section is ultraviolet divergent, and requires renormalization, which makes W_{DY} a function of renormalization scale μ. As discussed in [21], for instance, W_{DY} takes into account the contribution of soft gluons to the Drell-Yan cross section. The value of μ is arbitrary but the most convenient choice is $\mu \sim Q$. Let us show that for $\mu = Q$ the expression (11) indeed coincides with the resummation factor (8) up to power corrections in b^2.

Expanding (11) in powers of the gauge field, we get the following unrenormalized expression for W_{DY} to the lowest order of perturbation theory:

$$W_{\mathrm{DY}}^{(1)} = 1 + g^2 \mu^{4-D} C_F \int \frac{\mathrm{d}^D k}{(2\pi)^D} 2\pi \delta_+(k^2) \left(\frac{p_1}{p_1 \cdot k} - \frac{p_2}{p_2 \cdot k}\right)^2 (1 - \mathrm{e}^{-i\mathbf{k}_t \cdot \mathbf{b}})\,,\qquad(13)$$

where μ is the parameter of dimensional regularization, with $D = 4 - 2\varepsilon$, $\mathrm{d}^D k \equiv \mathrm{d}k^+ \mathrm{d}k^- \mathrm{d}^{D-2} k_t$, and $\mathbf{k}_t$ is the $(D-2)$-dimensional transverse momentum of the gluon. For massless quarks, the $k^\pm$ integral contains a collinear divergence. This divergence is eliminated, however, when one evaluates the logarithmic derivative $\mathrm{d}\ln W_{\mathrm{DY}}/\mathrm{d}\ln Q^2$. Using the one-loop expression $W_{\mathrm{DY}}^{(1)}$ and the exponentiation of infrared divergences in hard-scattering processes [4, 5, 27] we get

$$\frac{\mathrm{d}\ln W_{\mathrm{DY}}}{\mathrm{d}\ln Q^2} = 4 C_F \mu^{4-D} \int \frac{\mathrm{d}^{D-2} k_t}{(2\pi)^{D-2}} \frac{\alpha_s}{k_t^2} \left(\mathrm{e}^{-i\mathbf{k}_t \cdot \mathbf{b}} - 1\right).\qquad(14)$$

To take into account higher order corrections to (14) we choose the argument of the coupling constant to be the transverse momentum, $\alpha_s = \alpha_s(k_t^2)$, and identify $C_F \alpha_s/\pi$ as the lowest term in the expansion of the cusp anomalous dimension, $\Gamma_{\mathrm{cusp}}(\alpha_s(k_t^2))$ [4, 5],

$$\frac{\mathrm{d}\ln W_{\mathrm{DY}}}{\mathrm{d}\ln Q^2} = 4\pi \mu^{2\varepsilon} \int \frac{\mathrm{d}^{2-2\varepsilon} k_t}{(2\pi)^{2-2\varepsilon}} \frac{\Gamma_{\mathrm{cusp}}(\alpha_s(k_t^2))}{k_t^2} \left(\mathrm{e}^{-i\mathbf{k}_t \cdot \mathbf{b}} - 1\right).\qquad(15)$$

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Notice that after integration over large k_t^2 this expression contains ultraviolet poles in ε which need to be subtracted. The resulting renormalized expression contains both logarithmic and power corrections in b^2 to the eikonalized cross section, due to soft gluon emissions from the incoming quarks. Neglecting power corrections to (15), we find by comparison with eq. (5), that the behavior of $\ln W_{\mathrm{DY}}$ in $\ln Q$ in (15) is the same as the behavior of $S(Q, b)$ in $\ln Q$ up to nonleading logarithmic terms due to the "collinear" anomalous dimension Γ in (5). Identification of the resummation factor $\exp(-S(Q^2, b^2))$ as a Wilson line expectation value (15) allows us now to analyze the perturbative contribution of soft gluons with small transverse momentum k_t^2 to the Drell-Yan cross section.

In the evolution equation (15) and its solution W_{DY} we find divergences at $k_t^2 = \Lambda^2$. Notice that (15) was found after resummation of soft gluons to all orders of perturbation theory, and that this singularity can easily be translated into the high-order behavior of the original perturbative series before resummation. We can reconstruct the expansion in $\alpha_s(Q^2)$ by using the one (or higher) loop running coupling, $\alpha_s(k_t^2) = \alpha_s(Q^2)/[1 - \beta_1 \alpha_s(Q^2) \ln Q^2/k_t^2]$. Such an expansion generates a sum of integrals of the form $\int_0^1 dx \ln^n(1/x) = \Gamma(n+1)$, and we find that the singularity manifests itself in the perturbative expansion in $\alpha_s(Q^2)$ as an infrared renormalon in the exponent [13], behaving at nth loop order as $\alpha_s^n \beta_1^n n!$.

Let us now study in more detail how IR renormalons appear in the evolution equation (14), including renormalization. After substitution of the relation $\alpha_s(k_t^2) = \int_0^\infty d\sigma (k_t^2/\Lambda^2)^{-\sigma\beta_1}$ into (14), and after integration over transverse momenta, we get

$$\frac{d \ln W_{\mathrm{DY}}}{d \ln Q^2} = -\frac{C_F}{\pi}(\pi\mu^2 b^2)^\varepsilon \int_0^\infty \frac{d\sigma}{\sigma\beta_1 + \varepsilon} \frac{\Gamma(1 - \sigma\beta_1 - \varepsilon)}{\Gamma(1 + \sigma\beta_1)} \left(\frac{\Lambda^2 b^2}{4}\right)^{\sigma\beta_1}. \tag{16}$$

We notice that $(\Lambda^2 b^2/4)^{\sigma\beta_1} = e^{-\sigma/\alpha_s}$ with $\alpha_s = \alpha_s(4/b^2)$ and identify the right-hand side of the last relation as the Borel representation,

$$\pi(\alpha_s) = \int_0^\infty d\sigma \tilde{\pi}(\sigma) \, e^{-\sigma/\alpha_s}, \tag{17}$$

of $\pi(\alpha_s) \equiv d \ln W_{\mathrm{DY}}/d \ln Q^2$, with

$$\tilde{\pi}(\sigma) = -\frac{C_F}{\pi}(\pi\mu^2 b^2)^\varepsilon \frac{1}{\sigma\beta_1 + \varepsilon} \frac{\Gamma(1 - \sigma\beta_1 - \varepsilon)}{\Gamma(1 + \sigma\beta_1)}. \tag{18}$$

Let us analyze the singularities of the Borel transform $\tilde{\pi}(\sigma)$ for real positive σ. In the limit $\varepsilon \to 0$ the function $\tilde{\pi}(\sigma)$ has a pole for $\sigma = 0$. Since small values of the Borel parameter correspond to small coupling constants $\alpha_s(k_t^2)$, or equivalently to large transverse momenta k_t^2, the singularity at $\sigma = 0$ is ultraviolet. Away from $\sigma = 0$ we put $\varepsilon = 0$ and find that the function $\tilde{\pi}(\sigma)$ has singularities generated by $\Gamma-$functions at $\sigma^* = 1/\beta_1$, $2/\beta_1$, $3/\beta_1$, ..., the infrared renormalons. To define the integral over large σ we have to fix the prescription for integration of the IR renormalons in (16). Different prescriptions lead to results that differ from each other in powers of $b^2\Lambda^2$, beginning at $\mathcal{O}(b^2\Lambda^2)$, the contribution of the "leading" IR renormalon, at $\sigma^* = 1/\beta_1$.

To get a "perturbative" approximation to W_{DY}, we treat σ as a small parameter and then integrate (16) over $0 < \sigma < 1/2\beta_1$. For small σ we replace the ratio of $\Gamma-$functions in (16) by $e^{2\gamma\sigma\beta_1 + \gamma\varepsilon}$, use the expansion $1/(\varepsilon + \sigma\beta_1) = 1/\varepsilon - \sigma\beta_1/\varepsilon^2 + \ldots$ and change variables to $x = \sigma\beta_1$ to get

$$\frac{d \ln W_{\mathrm{PT}}}{d \ln Q^2} = \frac{C_F}{\pi\beta_1} \sum_{n=1}^\infty (-)^n \frac{(2\pi\mu^2 b^2/C)^\varepsilon}{\varepsilon^n} \int_0^{1/2} dx \, x^{n-1}(\Lambda^2 b^2/C)^x, \tag{19}$$

with C given in eq. (5). Subtracting poles in the $\overline{\mathrm{MS}}$ scheme, we find after summation the following result, for $\mu^2 = 2Q^2$ and $1/b^2 \gg \Lambda^2$,

$$\frac{\mathrm{d}\ln W_{\mathrm{PT}}}{\mathrm{d}\ln Q^2} = \frac{C_F}{\pi\beta_1} \int_{1/\ln(Q^2/\Lambda^2)}^{1/\ln(C/\Lambda^2 b^2)} \frac{\mathrm{d}x}{x} + \mathcal{O}(1) = \frac{C_F}{\pi\beta_1} \ln\frac{\ln(C/\Lambda^2 b^2)}{\ln(Q^2/\Lambda^2)} + \mathcal{O}(1) \,. \tag{20}$$

One easily checks that the resummation factor (8) satisfies this relation up to nonleading logarithms in Q, so that to this approximation, $W_{\mathrm{PT}} = \exp\left(-S(Q^2, b^2)\right)$.

There is no unique way of defining the k_t integral in eq. (15) or the σ integral in eq. (16). As discussed above, however, we may consider the singularities of the running coupling as inducing an *ambiguity* in the integral, which is to be eliminated by adding new, nonperturbative parameters to the theory [8, 28]. This process is not utterly arbitrary, because the singularity appears only at small values of k_t, where, unless b is very large (*i.e.*, of order $1/\Lambda$), the integrand is suppressed. This means that we may consistently define the full integral in the right-hand side of (15) as a power series expansion in b, starting with a "perturbative" contribution $S_{\mathrm{PT}}(b) = -\ln W_{\mathrm{PT}}(b)$, eq. (20), at order $(b^2)^0$:

$$e^{-S(Q^2, b^2)} \approx W_{\mathrm{DY}}(b) = \exp\left(-S_{\mathrm{PT}}(b) - b^2 S_2(Q) - b^4 S_4(Q) + \ldots\right) \,, \tag{21}$$

where the equality is approximate, since we have neglected nonleading collinear corrections to $S(Q^2, b^2)$. The fact that the Borel transform (18) has IR renormalon singularities at $\sigma^* = n/\beta_1$ for $n = 1, 2, \ldots$ implies that nonperturbative effects should contribute $\mathcal{O}(b^{2n}\Lambda^{2n})$ power corrections to the exponent in (21), or equivalently to the functions $S_{2n}(Q)$. Moreover, the explicit form of the evolution equation (15) implies that in the large Q limit these functions behave as

$$S_{2n}(Q) \sim A_{2n} \ln Q + B_{2n} \,. \tag{22}$$

This is because the IR renormalons arise from small values of k_t in (15), where we may take $\epsilon = 0$. Then the contribution of the IR renormalons in (15) to $\mathrm{d}W_{\mathrm{DY}}/\mathrm{d}Q^2$ is independent of Q, which, along with (21), implies the functional dependence (22).

3. Operator content of the leading IR renormalon

We now identify the operator content of the leading infrared renormalon, $\sigma^* = 1/\beta_1$, which contributes to the $b^2\Lambda^2$ correction to the right-hand side of (15), by recognizing it as the first term in the expansion of $\ln W_{\mathrm{DY}}$ with respect to b^2. We denote this contribution as

$$S_2(Q) = -\frac{\partial}{\partial b^2} \ln W_{\mathrm{DY}}(b)\bigg|_{b=0} \,. \tag{23}$$

Applying this derivative to the logarithm of W_{DY}, eq. (11), evaluated at $b = 0$, we find

$$S_2(Q) = \frac{1}{4}\langle 0| U_{DY}^\dagger(0)\, (i\vec{\partial})^2\, U_{DY}(0)\, |0\rangle \,, \tag{24}$$

where the derivative $\vec{\partial} = (\partial_1, \partial_2)$ acts in the transverse $b-$space. Finally, using the definition (10) of U_{DY} we get a form reminiscent of (13),

$$S_2(Q) = \frac{1}{4}\langle 0| \left| \Phi_{p_2}^\dagger(0, -\infty)\left(\vec{\mathcal{F}}_{p_1}(0) - \vec{\mathcal{F}}_{p_2}^\dagger(0)\right)\Phi_{p_1}(0, -\infty)\right|^2 |0\rangle \,, \tag{25}$$

where $\mathcal{F}_p^\alpha(x) \equiv (iD^\alpha\Phi_p(x,-\infty))\Phi_p^\dagger(x,-\infty)$ is a nonlocal "eikonalized" field strength, which may also be written in terms of the local field strength $F^{\mu\nu}$ and the ordered exponential Φ_p, eq. (9), as

$$\mathcal{F}_p^\alpha(x) = -ig \int_{-\infty}^0 ds\; \Phi_p(x, x+sp)\, p_\mu F^{\mu\alpha}(sp+x)\, \Phi_{-p}(x+ps, x)\,. \tag{26}$$

The perturbative contributions to S_2 are scaleless integrals and vanish in dimensional regularization. This is similar to the perturbative renormalization of the divergent matrix elements of a local operator such as $\langle 0|F^2|0\rangle$. Similarly, we may take $W_{\mathrm{DY}}(0)$ as unity.

From (25) and (26), we see that the leading nonperturbative correction is described by the matrix element of a nonlocal operator of the gauge field. This is in contrast with the total e^+e^- cross section, in which the leading IR renormalon appears at $\sigma = 2/\beta_1$ and gives rise to power corrections described by the local operator $\langle 0|F^2|0\rangle$. Nevertheless, it is possible to represent (25) as a local operator in an effective field theory. In this model, the quark with light-cone momentum p is described by the effective field, $q_p(x)$, and the interaction with gauge fields, $A_\mu(x)$, is organized to reproduce the quark-gluon interaction in the eikonal approximation. The Lagrangian of the model is given by

$$\mathcal{L} = \bar{q}_p(x)(p\cdot iD)q_p(x)\,, \tag{27}$$

where $\bar{q}_p = q_p^\dagger$, with p the quark momentum, and where $D_\mu = \partial_\mu - igA_\mu(x)$ is the covariant derivative in the quark representation. For a heavy quark with mass M, velocity v_μ and momentum $p = Mv$, this is the Lagrangian of heavy quark effective theory. The solution of the equation of motion for the $q-$field reproduces the eikonal phases of the quark fields

$$q_p(x) = \Phi_p(x,-\infty)\, a_p\,, \tag{28}$$

where $a_p = q_p(-\infty)$ is the creation operator of the eikonalized quark. Using this nonabelian "Bloch-Nordsieck" model, we may rewrite $U_{\mathrm{DY}}(x)$, eq. (10), as a composite local operator in the effective theory, $U_{\mathrm{DY}}(x) \sim \bar{q}_{p_2}(x)q_{p_1}(x)$, where fields q_{p_1} and $\bar{q}_{p_2}$ describe a quark and an antiquark with momenta p_1 and p_2, respectively. The fields q carry color indices of the quarks, and the summation over these indices is implied. In these terms, the matrix element in (24) is given by

$$E_2(Q) = \frac{1}{4}\langle 0|(\bar{q}_{p_1}q_{p_2})\,(i\vec{\partial})^2\,(\bar{q}_{p_2}q_{p_1})|0\rangle\,, \tag{29}$$

provided that $\langle 0|a_{p_1}^\dagger a_{p_1}|0\rangle = \langle 0|a_{p_2}^\dagger a_{p_2}|0\rangle = 1$. As was pointed out above, the value of this nonperturbative matrix element will depend on how we define perturbation theory. One definition is to apply a principal value prescription for integration around the poles in the inverse Borel transform, eq. (16) [13].

A different method is implicit in the prescription given by Collins and Soper [2], in which the impact parameter b is replaced by the function

$$b^* = \frac{b}{[1 + b^2 Q_0^2]^{1/2}}\,, \qquad Q_0 \sim 2\text{ GeV}\,, \tag{30}$$

in the expression for the exponent $S(Q^2, b^2)$ and for the quark distribution functions $f_{a/h}(x, C/b^2)$ in eq. (4), while an additional nonperturbative factor

$$\exp\left(-g_1(b^2,\tau) - g_2(b^2)\ln\frac{Q}{2Q_0}\right) \tag{31}$$

is introduced into the exponent S in eq. (5) to suppress the large-b region in the Fourier integral in eq. (3). In the large-b limit, b^* approaches a maximum value of $1/Q_0$ and the perturbative k_t^2 integral in (5) becomes well-defined. For moderate $1/Q \ll b \ll 1/\Lambda$, the following parameterization was also proposed, again consistent with the result of eq. (21) above,

$$g_1(b^2, \tau) = g_1 b^2 + \mathcal{O}(b^4), \qquad g_2(b^2) = g_2 b^2 + \mathcal{O}(b^4). \tag{32}$$

The values of g_1 and g_2 cannot be calculated in perturbative QCD, but have been estimated [12] from experiment[3],

$$g_1 = 0.15 \ (\text{GeV})^2, \qquad g_2 = 0.40 \ (\text{GeV})^2 \, .$$

In summary, the results (21) and (22) of our analysis are in agreement with the Collins-Soper parameterization of nonperturbative effects in (31) and (32). In addition, our considerations give a field-theoretic interpretation to the coefficient g_2, as

$$g_2 = \frac{\mathrm{d}S_2(Q)}{\mathrm{d}\ln Q} = A_2 \tag{33}$$

with S_2 and A_2 given in (25) and (22). Of course, the specific value of g_2 quoted above depends on the Collins-Soper method, (30) and (31), of defining the perturbative content of resummation. Other values will give different g_2's in general, an ambiguity that we expect from general considerations.

It is now useful to discuss the nature of our approximations. We have taken into account the contributions of soft gluon radiation from active quarks, and have ignored the contributions of collinear gluons as nonleading in $\ln Q^2$. In perturbation theory, this property follows from the expression (5) for the resummation factor, in which purely collinear interactions contribute to the anomalous dimensions Γ, while soft emissions contribute to both Γ and Γ_{cusp}. Generalizing this observation to the power corrections in the exponent S, eq. (21), we find that collinear interactions modify the coefficients B_{2n} in (22), but not the leading coefficients A_{2n}. In addition, considering the expression (4) for $\tilde{\Sigma}(Q^2, b^2)$, we notice that for large b^2 the normalization scale in the definition of the quark distribution functions becomes small, which should produce nonperturbative power corrections to $f_{a/h}(x_a, C/b^2)$. Such corrections are naturally attributed to "intrinsic" motion of quarks inside hadrons. This motion affects the transverse momentum of the lepton pair, and may be parameterized phenomenologically by the function $g_1(b^2, \tau)$ in (31). This implies that the coefficients B_{2n} in (22) reflect the intrinsic momenta of quarks in hadrons [4]. In realistic hadronic states, we expect quarks to have a "residual" virtuality, which should replace $1/b^2$ as a regulator of collinear divergences in $S(Q^2, b^2)$, (5), for large b^2. At the same time, the coefficients A_{2n} are free from collinear divergences, i.e., are infrared safe, and are thus independent of quark virtuality. Therefore, we expect the coefficients A_{2n} to be independent of the incoming hadrons. This explains why only the vacuum expectation values of operators appear in relations (11) and (25).

It is clear that this pattern will recur whenever a cross section can be written in terms of a resummed expression like eq. (5), that involves the integral over the scale of the running coupling. In the following two sections, we shall find two other applications, jet cross sections and the inclusive dilepton cross section, which show a similar pattern, and which illustrate the dependence of the implied nonperturative corrections on the process.

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[3]Another parameterization was proposed in [29].

[4]Similar phenomena occur in the behavior of the heavy quark distribution function in the end-point region [15].

4. Power corrections in jet cross sections

As another application of our infrared renormalon analysis, we examine power corrections to infrared-safe jet cross sections. Let us consider the cross section for $e^+e^- \to 2$ jets, defined by calorimeters in the form of back-to-back cones of half-angle δ in the overall center of mass. Let $\sqrt{s} = Q$ be the center of mass energy, and let xQ be the total energy flowing into the two jets. For $x \to 1$, the cross-section $d\sigma_{2J}/dx$ receives large perturbative corrections associated with the emission of soft gluons outside the jet cone, with total energy $(1-x)Q$.

As in the Drell-Yan process, the asymptotic behavior of the cross section $d\sigma_{2J}/dx$ for $x \to 1$ is best seen in terms of its Fourier transform with respect to the total energy $(1-x)Q$ carried by soft gluons,

$$\frac{d\sigma_{2J}((1-x)Q, \delta)}{dx} = \int_{-\infty}^{\infty} \frac{dy_0}{2\pi} \, e^{-iy_0 Q(1-x)} \, \tilde{\sigma}(y_0, \delta) \,. \tag{34}$$

For large y_0 (conjugate to $x \to 1$), large perturbative corrections to the Fourier transform $\tilde{\sigma}(y_0, \delta)$ exponentiate in much the same way as for the transverse momentum distribution in the Drell-Yan process discussed above, due to the factorization of soft gluons from the two energetic jets. As in the previous case, we neglect the nonleading contributions of collinear gluons, and concentrate on the leading behavior in δ only, given by

$$\tilde{\sigma}(y_0, \delta) = \exp(-S_{2J}(y_0, \delta)) \,. \tag{35}$$

To lowest order the exponent S_{2J} is given by (compare (13)),

$$S_{2J}(y_0, \delta) = g^2 C_F \int \frac{d^D k}{(2\pi)^D} \, \theta_{\text{jet}}(k) \, 2\pi \delta_+(k^2) \left(\frac{p_1}{p_1 \cdot k} - \frac{p_2}{p_2 \cdot k} \right)^2 (1 - e^{ik_0 y_0}) \,, \tag{36}$$

with $p_1 = \frac{Q}{2}(1, 1, \mathbf{0})$ and $p_2 = \frac{Q}{2}(1, -1, \mathbf{0})$ the light-like momenta of quarks in the two energetic jets. The function $\theta_{\text{jet}}(k)$ is zero when k is in a jet cone, and unity otherwise. Making explicit this kinematical restriction on the momenta of soft gluons, and including higher order corrections to $S_{2J}(y_0, \delta)$, we get, to leading logarithmic accuracy,

$$S_{2J}(y_0, \delta) = -2 \int_0^Q \frac{dk_0}{k_0} \left(1 - e^{ik_0 y_0}\right) \int_{k_0^2 \delta^2}^{k_0^2} \frac{dk_t^2}{k_t^2} \Gamma_{\text{cusp}}(\alpha_s(k_t^2)) \,, \tag{37}$$

where integration is performed over energy and over transverse momentum with respect to the axes of the outgoing jets. The soft gluon energy in (37) is restricted to be less than the energy of the "parent" quark, while the lower limit, $k_t > k_0 \delta$, implies that the soft gluon is emmitted outside the jet cone. After substitution of (37) and (35) into (34) and integration over y_0, eq. (34) reproduces the leading perturbative series for the eikonalized 2-quark jet cross section with the total energy of gluons outside the cones given by $(1-x)Q$.

To take into account higher order corrections to the cross section, we have chosen, as usual, the argument of the coupling constant to be k_t^2. An evolution equation for the exponent, $S_{2J}(y_0, \delta)$, now follows from (37) (compare eq. (15)) in terms of the "cone" parameter δ,

$$\frac{dS_{2J}(y_0, \delta)}{d \ln \delta} = 4 \int_0^Q \frac{dk_0}{k_0} \left(1 - e^{ik_0 y_0}\right) \Gamma_{\text{cusp}}(\alpha_s(k_0^2 \delta^2)) \,. \tag{38}$$

On the right-hand side of this equation, an infrared renormalon arises once again, from integrations over soft gluon energy $k_0 \sim \Lambda/\delta$, due to the singularity in $\alpha_s(k_0^2 \delta^2)$. Taking the one-loop expression (6) for the cusp anomalous dimension and using the representation $\alpha_s(k_0^2 \delta^2) =$

$\int_0^\infty d\sigma(k_0^2\delta^2/\Lambda^2)^{-\sigma\beta_1}$, we may perform the integration over k_0 in (38) to get an expression similar to (16). However, an important difference from (16) is that the IR renormalons now appear at $\sigma^* = 1/2\beta_1, 1/\beta_1, 3/2\beta_1, \dots$. Thus, in contrast with the transverse momentum distribution, the leading IR renormalon appears in the $e^+e^- \to 2$ jets cross section at $\sigma^* = 1/2\beta_1$, which gives rise to an $\mathcal{O}(y_0\Lambda/\delta)$ power correction to the exponent, $S_{2J}(y_0, \delta)$ in (35). We conclude that in the jet cross section, $\tilde{\sigma}(y_0, \delta)$, the leading nonperturbative power corrections occur at level $y_0\Lambda/\delta$.

Now suppose we regulate the exponent $S_{2J}(y_0, \delta)$ by some procedure, for example, a y_0-dependent cutoff, by analogy to the Collins-Soper procedure in (30) and (31), or a principal value prescription for integrating over IR renormalons. We denote the result by $S_{2J,PT}(y_0, \delta)$. Taking into account the contribution of the leading IR renormalon to $S_{2J}(y_0, \delta)$, we may write the cross section as

$$\tilde{\sigma}(y_0, \delta) = \exp\left(-S_{2J,PT}(y_0, \delta) - i\frac{y_0\Lambda}{\delta}A_{2J} + \mathcal{O}(y_0^2)\right),\tag{39}$$

with A_{2J} a new dimensionless nonpertubative parameter. In place of eq. (34), we now have

$$\frac{d\sigma_{22J}(xQ, \delta)}{dx} = \int_{-\infty}^\infty \frac{dy_0}{2\pi}e^{-iy_0Q(1-x+A_{2J}\Lambda/Q\delta)}\ \exp\left(-S_{2J,PT}(y_0, \delta)\right).\tag{40}$$

As usual, A_{2J} is dependent on our definition of $S_{2J,PT}$. We thus see that the effect of the leading infrared renormalon is to shift the scaling variable x by an amount inversely proportional to the typical transverse momentum in the jets, $Q\delta$.

An operator intepretation of this result is not as simple as for the examples discussed in the previous section, because eq. (37) requires restrictions on both angles and energies of the soft gluons that give double logarithms. Nevertheless, the same leading logarithmic corrections can be generated by introducing a product of outgoing massive eikonal lines, defined as in eq. (10), but with

$$p_1^2 = p_2^2 = m^2\,, \qquad m^2 = Q^2\delta^2\,.\tag{41}$$

The corresponding operator which takes into account soft gluon emissions into the final state is now

$$U_{2J}(x) = T\left[\Phi_{p_1}(\infty, x)\Phi_{p_2}^\dagger(\infty, x)\right],\tag{42}$$

and a corresponding cross section for the emission of gluons of total energy $(1-x)Q$ is given by (34) with

$$\tilde{\sigma}(y_0, \delta) = W_{e^+e^-} \equiv \langle 0|U_{2jet}^\dagger(0)U_{2jet}(y)|0\rangle\,,\tag{43}$$

where $y^\mu = (y_0, 0, \mathbf{0})$. Indeed, eq. (43) has the same perturbative expansion as (35) and (36), except that now the quarks have time-like momenta, $p_1 = (E, p, \mathbf{0})$ and $p_2 = (E, -p, \mathbf{0})$ with $E^2 = p^2 + m^2$, and the gluon momentum integrals are unrestricted. Then, integrating over k_3 we get the following leading logarithmic expression for the exponent $S_{2J}(y_0, \delta)$,

$$S_{2J}(y_0, \delta) = -2\int_0^\infty \frac{dk_0}{k_0}\left(1 - e^{ik_0y_0}\right)\int_{k_0^2m^2/E^2}^{k_0^2}\frac{dk_t^2}{k_t^2}\Gamma_{cusp}(\alpha_s(k_t^2))\,,\tag{44}$$

where $E \approx Q/2$ is the quark energy. Due to the cusp singularities of the Wilson lines, the k_0−integral in (44) is ultraviolet divergent and should be regularized, for example, dimensionally, as in eq. (14). The net effect of the regularization is to introduce a cut-off μ for large energies

k_0. Taking $\mu = Q$ and identifying the mass of the quark as in (41), we find that (44) coincides with expression (37) for $S_{2J}(y_0, \delta)$. Clearly, to incorporate nonleading logarithms, which depend, in particular, on the internal structure of the jet, we must go beyond this rather crude approximation. Besides these nonleading terms, there are additional corrections in this model associated with an ambiguity in the definition of the mass of eikonalized quarks. As discussed in [30], the mass itself suffers from an ultraviolet renormalon, which introduces an ambiguity of order Λ into its definition. The corresponding ambiguity in the final-state energy contributes a term of order Λy_0 to the exponent $\exp[-iy_0 Q(1 - x + A_{2J}\Lambda/Q\delta)]$ in eq.(40), down by a factor of δ from the contribution we have identified.

Since the first infrared renormalon singularity is proportional to y_0, we may identify the parameter A_{2J} with the first derivative of $W_{e^+e^-}(y_0, \delta)$ with respect to y_0 at $y_0 = 0$, by analogy to eq. (25) above, and we find that

$$A_{2J} = \langle 0|\bar{\mathrm{T}}\left[\Phi_{p_1}(0, \infty)\left(\mathcal{F}_{0p_1}(0) - \mathcal{F}_{0p_2}^\dagger(0)\right)\Phi_{p_2}^\dagger(0)\right]^\dagger \mathrm{T}\left[\Phi_{p_1}(0)\Phi_{p_2}^\dagger(0)\right]|0\rangle. \tag{45}$$

Note that this form involves only a single factor of the field strength, made gauge invariant by its combination with path-ordered exponentials.

One may be tempted to suggest that such a matrix element, with only a single field-strength, must vanish. In fact, if we were allowed to remove the time and anti-time ordering from (45), this would be the case. That is, we could use the property $\Phi_p^\dagger \Phi_p = 1$ of ordered exponentials to simplify (45) to $\langle 0|\left(\mathcal{F}_{0p_1}(0) - \mathcal{F}_{0p_2}^\dagger(0)\right)|0\rangle$. This matrix element, however, is zero, because $\langle 0|\mathcal{F}_{\mu,p}|0\rangle = cp_\mu$, with c some constant, while $p^\mu\langle 0|\mathcal{F}_{\mu p}|0\rangle = \langle 0|(p \cdot D)\Phi_p\Phi_p^\dagger|0\rangle = 0$ from the definition of the Wilson line, which implies that $c = 0$. This argument fails, however, because the gauge fields on the p_1 and p_2 eikonal lines do not commute in general. Thus, the matrix element in (45) need not be zero. As usual, its precise value will depend on how we define the ambiguity in the resummed perturbation series. The important point here is that the matrix element depends on a single, although nonlocal, vacuum matrix element. A more detailed study of the large variety of jet cross sections will result in further information.

5. Power corrections in inclusive lepton-pair production

As final example, we consider the inclusive Drell-Yan cross section $\mathrm{d}\sigma_{\mathrm{DY}}(\tau, Q^2)/\mathrm{d}Q^2$, normalized to a structure function of deeply inelastic scattering, $F_{\mathrm{DIS}}(x, Q^2)$ [21, 22, 23, 24, 13]. To be specific, the moments of the dilepton hard-scattering function are related to those of the DIS structure functions by

$$\int_0^1 \mathrm{d}\tau\, \tau^{n-1}\frac{\mathrm{d}\sigma_{\mathrm{DY}}(\tau, Q^2)}{\mathrm{d}Q^2} = \sigma_0\, \omega(n, Q^2)\, F_{\mathrm{DIS}}^2(n, Q^2), \tag{46}$$

where $\tau = Q^2/s$ and σ_0 is the inclusive Born cross section, as in eq. (1), and where $F_{\mathrm{DIS}}(n, Q^2)$ is the corresponding moment of $F_{\mathrm{DIS}}(x, Q^2)$ with respect to x. For large values of n (conjugate to $\tau \to 1$ and $x \to 1$), $\omega(n, Q^2)$ contains large perturbative corrections of the form $\alpha_s^k \ln^{2k} n$.

The logarithms of n, however, are readily resummed. In fact, up to corrections that are bounded in n, the moments of the hard-scattering function exponentiate in the form

$$\omega(n, Q^2) = R(\alpha_s(Q^2)) \exp\left[E(n, Q^2)\right], \tag{47}$$

where $R(\alpha_s)$ is a finite function of α_s, while the function $E(n, Q^2)$ generates all logarithmic corrections in n from the integrals[5]

$$E(n, Q^2) = -2 \int_0^1 dz \, \frac{z^{n-1} - 1}{1 - z} \left[\int_{(1-z)^2 Q^2}^{(1-z)Q^2} \frac{dk_t^2}{k_t^2} \Gamma_{\text{cusp}}(\alpha_s(k_t^2)) + \Gamma\left(\alpha_s((1-z)Q^2)\right) \right] . \tag{48}$$

The anomalous dimensions Γ_{cusp} and Γ have been encountered above in eq. (6). $E(n, Q^2)$ contains infrared renormalon singularities from small values of k_t. We may isolate the leading infrared renormalon by interchanging the k_t and z integrals in the first term of (48),

$$\begin{aligned} E(n, Q^2) &= -2 \int_0^{Q^2} \frac{dk_t^2}{k_t^2} \Gamma_{\text{cusp}}(\alpha_s(k_T^2)) \int_{1 - k_t/Q}^{1 - k_t^2/Q^2} dz \, \frac{(1 - (1 - z))^{n-1} - 1}{1 - z} \\ &= 2(n - 1) \int_0^{Q^2} \frac{dk_t^2}{k_t^2} \Gamma_{\text{cusp}}(\alpha_s(k_t^2)) \left[\frac{k_T}{Q} - \frac{k_T^2}{Q^2} \left(\frac{n - 2}{4} \right) \right] + \cdots , \end{aligned} \tag{49}$$

where we have expanded z^{n-1} in powers of $1 - z$, and have suppressed $(1 - z)^3$ and higher. Such terms produce higher powers of nk_t/Q. For large values of k_t, all such terms contribute to the perturbative expansion of $E(n, Q^2)$. The leading infrared renormalon, however, which may be identified in the same manner as above in eq. (16), is present only in the first term in the expansion of (49), which we denote

$$\frac{4nA_{\text{DY}}(Q)}{Q} \equiv 4\frac{n}{Q} \int_0^{Q^2} dk_T \, \Gamma_{\text{cusp}}(\alpha_s(k_T^2)) . \tag{50}$$

Again, $A_{\text{DY}}(Q)$ includes both perturbative and nonperturbative contributions. The latter is precisely the $1/Q$ contribution identified in [13]. Following our procedure above, we now find a nonlocal matrix element which corresponds to $A_{\text{DY}}(Q)$.

We begin by defining a product of Wilson lines that reproduces the logarithmic behavior of the Drell-Yan cross section near the boundary of phase space [24],

$$W_{\text{DY}}(k_0, \mu) = \int \frac{dy_0}{2\pi} e^{-ik_0 y_0} \langle 0| U_{\text{DY}}^\dagger(0) \, U_{\text{DY}}(y)|0\rangle , \tag{51}$$

with U_{DY} the product of ordered exponentials in the directions of the incoming particles, given above in eq. (10) for $x = y \equiv (y_0, \mathbf{0})$. As indicated, W_{DY} requires renormalization at the scale μ. From the procedure introduced in [21], W_{DY} satisfies an evolution equation whose solution is the convolution form[6],

$$\begin{aligned} W_{\text{DY}}(k_0, \mu) &= \tilde{R}(\alpha_s, \varepsilon) \sum_{n=0}^{\infty} \frac{2^n}{n!} \prod_{i=1}^{n} \int_0^1 dz_i \left[\frac{1}{1 - z_i} \int_0^{(1-z_i)\mu} \frac{dk_{t,i}^2}{k_{t,i}^{2+2\varepsilon}} \Gamma_{\text{cusp}}(\alpha_s(k_{t,i}^2)) \right. \\ &\quad \left. + \tilde{\Gamma}\left(\alpha_s((1 - z)\mu^2)\right) \right]_+ \delta\left(k_0 - \sum_1^n (1 - z_i)\mu \right) . \end{aligned} \tag{52}$$

Here, $\tilde{R}$ is an (infrared sensitive) function of α_s and ε, and $\tilde{\Gamma}$ organizes nonleading logarithms, which may differ from those in $E(n, Q^2)$ above. We may now evaluate the Fourier transform of W_{DY} explicitly,

$$\tilde{W}_{\text{DY}}(y_0, \mu) = \int \frac{dk_0}{2\pi} e^{ik_0 y_0} W_{\text{DY}}(k_0, \mu) . \tag{53}$$

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[5]In [22] and [23], $2\Gamma_{\text{cusp}}$ and 2Γ are denoted g_1 and g_2, respectively.
[6]The reasoning is essentially identical to that given in Section 5 of [21].

We then recognize that its derivative with respect to y_0, evaluated at $y_0 = 0$, generates the product of $\tilde{W}_{\mathrm{DY}}(0, \mu)$, which can be normalized to unity, and a single integral over k_t, in which appears in the scale of the running coupling,

$$- i \frac{\partial \ln \tilde{W}_{\mathrm{DY}}(y_0, \mu)}{\partial y_0}\Big|_{y_0=0} = 2 \int_0^{\mu^2} \frac{\mathrm{d}k_t^2}{k_t^{2+2\varepsilon}} \Gamma_c(\alpha_s(k_t^2)) \, (\mu - k_t) \, . \tag{54}$$

While both of the terms on the right-hand side contain an infrared renormalon, only the second corresponds to the function $A_{\mathrm{DY}}(\mu)$ identified from the resummed cross section above. To isolate $A_{\mathrm{DY}}(\mu)$ from the ultraviolet contribution in eq. (54), we simply form the combination,

$$A_{\mathrm{DY}}(\mu) = -\frac{i}{2} \frac{\partial \ln \tilde{W}_{\mathrm{DY}}(y_0, \mu)}{\partial y_0}\Big|_{y_0=0} - \frac{1}{\mu} \frac{\partial^2 \ln \tilde{W}_{\mathrm{DY}}(y_0, \mu)}{\partial y_0^2}\Big|_{y_0=0} \, . \tag{55}$$

Here, the second term, which is evaluated just as in (54), serves only to cancel the large, ultraviolet contribution. Its nonperturbative contribution, however, begins only at order $\mu^{-2} \sim Q^{-2}$, which we neglect in our approximation. Note that the second term is not unique, and higher derivatives (times higher powers of $1/\mu$) would equally well cancel the ultraviolet contribution, while contributing infrared renormalons that are yet further suppressed in $\mu \sim Q$.

6. Interpretation

We have studied several cross sections, whose leading logarithms are generated by products of ordered exponentials. In each case, we found that nonleading power corrections are required by the presence of infrared renormalons in the corresponding resummation formulas. These corrections, in turn, may be represented as vacuum matrix elements of field strengths, integrated over the paths of the original ordered exponentials. It is important to stress that our results do not depend on an extrapolation of the k_t^2 integral in, for example, eq. (5), to soft momenta, $k_t^2 \sim \Lambda^2$. Rather, eq. (5) organizes the large-order behavior of the cross section that follows from the evolution equation (15). In this sense, the presence of the divergence in the running coupling follows from the factorial behavior of perturbation theory, not the other way around.

In the specific cross sections we have studied, we have found that the first nonleading-Q behavior is associated with nonperturbative matrix elements, such as (25), (45) and (55). From our critereon of consistency for the full theory, including both perturbative and nonperturbative contributions, we conclude that these power corrections exponentiate, to give, for example, a nonperturbative Gaussian distribution in b. The Fourier transform of such a function gives a Gaussian behavior in momentum space, which must be convoluted with the perturbative Q_T distribution. On the other hand, behavior linear in the transform variable, such as n/Q in the Drell-Yan normalization and y_0/Q in the jet cross section, is associated with a shift in the conjugate kinematic variable.

The reasoning that leads to this picture is similar to that described in [8], which connected infrared renormalons in the total e^+e^- annihilation cross section with the local condensate $\langle 0|F^2(0)|0\rangle$. Here, we must go beyond the class of local operators to "nonlocal condensates", but with the benefit of greater flexibility, and perhaps, generality of application. Of course, this procedure lacks the guiding principles of the operator product expansion, and we have chosen to let perturbation theory suggest for us the form of nonperturbative structures that we may expect. While other contributions, not related to perturbation theory, are probably present,

the self-consistency of the theory demands the presence of those we identify from perturbative calculations.

We emphasize again the necessarily ambiguous nature of the magnitudes of the higher-twist matrix elements (e.g., (25)), which depend on the manner in which the perturbative integrals have been constructed and, indeed, the order to which they have been computed. In some sense, however, the situation is a bit better than for the total e^+e^- annihlation cross section, as described in Ref. [28], where beyond low orders, the perturbative series will begin to show $\Gamma(n)$ behavior. In resummation formulas, such behavior is already included with known coefficients. If we succeed, therefore, in constructing a perturbative exponent S whose (arbitrary) higher twist is not overly sensitive to higher-order corrections to (for instance) Γ_{cusp}, then the value of our nonperturbative matrix elements will correspondingly be stable to higher-order corrections in the perturbative calculation. Of course, there are other short-distance corrections not included in the resummation, but these will appear only at still higher twist.

Our discussion has been quite formal, but we hope that it will prove valuable to recognize that a limited set of nonperturbative parameters may enter into the first higher-twist corrections to many high energy cross sections. Such "universality" has proved of great practical use when applied to local condensates in QCD sum rules, despite the limitations and ambiguities inherent in their combination with perturbative calculations. Beyond this, were it possible to actually compute, by some nonperturbative method, highly nonlocal and relativistic operator combinations such as eq. (25), in a manner consistent with a particular perturbative construction of S, then the formalism presented here would provide new tests of the theory.

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